เฉลยเรื่อง คลื่น

(1) $y(x,t) = (0.15\ m)\sin(0.8x - 50t)$

(a) อัตราเร็วคลื่น $= \frac{\omega}{k} = v$

เทียบกับ $y(x,t) = A\sin(kx - \omega t)$

จะได้ $\omega = 50\ rad/s$

$k = 0.8\ m^{-1}$

$\Rightarrow v = \frac{\omega}{k} = \frac{50\ rad/s}{0.8\ m^{-1}} = 62.5\ m/s$ #

(b) ความยาวคลื่น, λ

จาก $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k}$

$\Rightarrow \lambda = \frac{2\pi}{0.8\ m^{-1}} = 7.85\ m$ #

(c) ความถี่คลื่น, f

จาก $f = \frac{\omega}{2\pi} = \frac{50}{2\pi}\frac{rad/s}{rad.} \Rightarrow$

$\Rightarrow f = 7.96\ s^{-1}$

$= 7.96\ Hz$ #

(d) กำลังที่ส่งออกไป $P = \frac{1}{2}\mu\omega^2 v A^2$

μ = มวลต่อความยาว

$\Rightarrow P = 1 \times 12\times10^{-3} \times 50\times50\times62.5\times(0.15)^2\ W$

② จาก $y(x,t) = (0.35\,m)\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$

ⓐ

$\omega = 10\pi$ rad/s

$k = 3\pi$ m^{-1}

$A = 0.35$ m

$\mu = 75 \times 10^{-3}$ kg/m

$$v = \frac{\omega}{k} = \frac{10\pi}{3\pi} \Rightarrow = \frac{10}{3}\ m/s$$

อัตราการส่งผ่านพลังงาน = กำลัง = $P = \frac{1}{2}\mu\omega^2 v A^2$

$$\Rightarrow P = \frac{1}{2} \times 75 \times 10^{-3} \times (10\pi)^2 \times \frac{10}{3} \times (0.35)(0.35)$$

$$= 15.1\ W$$

ⓑ พลังงานต่อรอบ $E_\lambda = PT$ [illegible]

$= (15.1)T$, แต่ $T = \frac{1}{f}$

$$f = \frac{\omega}{2\pi} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2\ s$$

$$E_\lambda = (15.1)(0.2) = 3.12\ J$$ #

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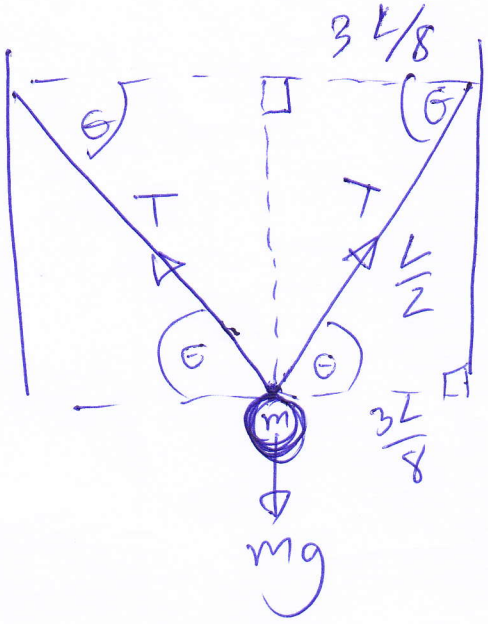
(3)



$$\sqrt{\frac{L^2}{4} - \frac{9L^2}{64}} = \sqrt{\frac{16L^2}{64} - \frac{9L^2}{64}} = \frac{\sqrt{7}L}{8}$$

(a)

On $V = \sqrt{\frac{T}{\mu}}$, $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta}$$

$$\text{Now } \sin\theta = \frac{\frac{\sqrt{7}L}{8}}{\frac{L}{2}} = \frac{\sqrt{7}}{4}$$

$$\Rightarrow T = \frac{mg}{2\frac{\sqrt{7}}{4}} = \frac{2mg}{\sqrt{7}}$$

$$\Rightarrow V = \sqrt{\frac{2mg}{\sqrt{7}\mu}} = \sqrt{\frac{2mg}{\sqrt{7}\times 8\times 10^{-3}}} \quad \#$$

(b)

$$\text{On } T = \frac{2mg}{\sqrt{7}} = v^2\mu$$

$$\Rightarrow m = \frac{\sqrt{7}}{2}\frac{v^2\mu}{g} = \frac{\sqrt{7}}{2}\times\frac{60\times 60\times 8\times 10^{-3}}{9.8}$$

$$= 3.89 \text{ kg} \quad \#$$

(4)

จาก $y(x,t) = 2A\sin kx \cos\omega t$

$$\frac{\partial y}{\partial t} = (2A\sin kx)(\omega)(-\sin\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = (2A\sin kx)(\omega)(-1)(\omega)\cos\omega t$$

$$= -\omega^2\, 2A\sin kx\cos\omega t$$

$$= -\omega^2 y \qquad \text{———} \ (1)$$

$$\frac{\partial y}{\partial x} = 2kA\cos kx\cos\omega t$$

$$\frac{\partial^2 y}{\partial x^2} = -2k^2 A\sin kx\cos\omega t$$

$$= -k^2[2A\sin kx\cos\omega t]$$

$$= -k^2 y \qquad \text{———} \ (2)$$

จาก (1) $\quad y = -\frac{1}{\omega}\frac{\partial^2 y}{\partial t^2} \qquad \text{———} \ (3)$

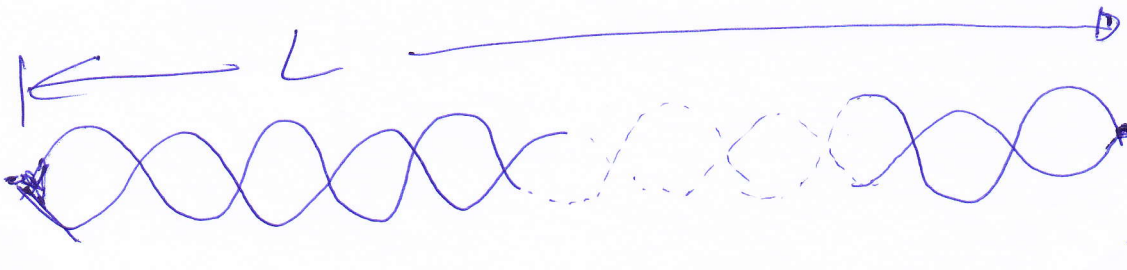
จาก (2) $\quad y = -\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2} \qquad \text{———} \ (4)$

(3) เท่ากัน (4) $\Rightarrow \quad -\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2} = -\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$

$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = \left(\frac{\omega}{k}\right)^2 \frac{\partial^2 y}{\partial x^2}$ แต่ $\frac{\omega}{k} = v$

$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \text{———} \ \#$

(5)



$T = mg$

จาก $L = n\frac{\lambda}{2}$

n คือ จำนวน loop

และ v = อัตราเร็วคลื่น $= \sqrt{\frac{T}{\mu}}$

ดังนั้น $L = n_1\frac{\lambda_1}{2} = n_2\frac{\lambda_2}{2}$

$T_1 = m_1 g$

$T_2 = m_2 g$

~~$v = f\lambda$~~ หรือ $v_1 = f\lambda_1$ และ $v_2 = f\lambda_2$

$$\sqrt{\frac{T_2}{T_1}} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2}$$

แทนค่า $T_1 = 16g$, $T_2 = 25g$

$$\sqrt{\frac{25g}{16g}} = \frac{n_1}{n_2} \Rightarrow \frac{n_1}{n_2} = \frac{5}{4}, \quad n_1 = 5,\ n_2 = 4$$

(a) ~~ค่า~~ $f = \frac{v_1}{\lambda_1} = \frac{\sqrt{\frac{T}{\mu}}}{\frac{2L}{n_1}} = \frac{n_1}{2L}\sqrt{\frac{T}{\mu}} = \frac{n_2}{2L}\sqrt{\frac{T_2}{\mu}}$

แทนค่า $f = \frac{5}{4}\sqrt{\frac{16\times 9.8}{0.002}} = 350\ Hz$

(b) จาก ~~$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$~~ ถ้า $n = 1$ แทนค่าลงในสมการ

เนื่องจาก $T = mg \propto \frac{1}{n}$

แทนค่า $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{mg}{\mu}}$

$f^2 = \frac{1}{4L^2}\frac{mg}{\mu} \Rightarrow m = \frac{4f^2L^2\mu}{g}$

$m = \frac{4\times 350\times 350\times 2\times 2\times 0.002}{9}$